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# On the Bayes Estimators of Parameter and Reliability Function of the Zero-Truncated Generalized Poisson Distribution

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**ABSTRACT**

In this paper Bayes estimator and reliability function of the zero-truncated generalized Poisson distribution (ZTGPPD) are obtained. Furthermore, recurrence relations for the estimator of the parameters are also derived. Monte Carlo simulation technique has been used for comparing the Bayes estimator and reliability function of ZTGPD with the corresponding maximum likelihood estimator (MLE).

***Keywords:*** Zero-truncated generalized Poisson distribution, reliability function, Bayes estimator, recurrence relation, Monte Carlo simulation, maximum likelihood estimation.

## I. INTRODUCTION

Consul and Jain (1973) defined the generalized Poisson distribution (GPD) as

$$P_1(X=x)=\frac{\lambda_1(\lambda_1+x\lambda_2)^{x-1}e^{-(\lambda_1+x\lambda_2)}}{x!},\ x=0,1,2,\ldots,\ \lambda_1>0,\ |\lambda_2|<1 \qquad (1.1)$$

It can be easily seen that at $\lambda_2 = 0$, the distribution (1.1) reduces to Poisson distribution. This model has been found to be a member of the Consul and Shenton's (1972, 1973) family of Lagrangian distributions and also of the Gupta's (1974) modified power series distribution (MPSD). Consul and Shoukri (1985, 1986) studied GPD when the sample mean is larger than the sample variance and for negative integer moments. Tuenter (2000) also discussed the GPD.

The problem of estimation for GPD has been considered by many authors. Whereas Consul and Shoukri (1984), Famoye and Lee (1992) and Consul and Famoye (1988, 1989) studied the maximum likelihood estimation. Gupta (1977) and Kumar and Consul (1980) discussed the minimum variance unbiased estimation. Shoukri and Consul (1989) and Hassan and Harman (2003) also studied the Bayes estimator for GPD using non-informative prior and more general prior gamma distribution.

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### 1.1 Zero Truncated Generalized Poisson Distribution (ZTGPD):

Consul and Shoukri (1989) redefined the distribution (1.1) by taking $\lambda_1 = \alpha$ and $\lambda_2 = \alpha\beta$ as:

$$P_2(X=x)=\frac{(1+\beta x)^{x-1}\alpha^x e^{-\alpha(1+\beta x)}}{x!}, \quad x=0,1,2,\cdots, \quad \alpha>0,\ 0<\beta<\frac{1}{\alpha} \tag{1.2}$$

The distribution (1.2) can be truncated at x = 0 and is defined as

$$P_3(X=x)=\frac{(1+\beta x)^{x-1}}{x!}\alpha^x e^{-\alpha(1+\beta x)}(1-e^{-\alpha})^{-1}, \ x=1,2,\cdots, \ \alpha>0,\ 0<\beta<\frac{1}{\alpha} \tag{1.3}$$

It can be easily seen that at $\beta = 0$, the distributions (1.2) and (1.3) reduce to Poisson distribution and to David and Johnson's (1952) truncated Poisson distribution with parameter.

Consul and Famoye (1989) defined the truncated generalized Poisson distribution (TGPD) and obtained its maximum likelihood estimation of the parameters based upon the mean and ratio of the first two frequencies. Jani and Shah (1981) also studied the truncated generalized Poisson distribution. Kyriakoussis and Papadopoulos (1993) discussed the Bayes estimators of the probability of success and reliability function of the zero-truncated binomial and negative binomial distributions. Bansal and Ganji (1997) discussed the Bayes estimation in a decapitated form of Consul and Jain's (1971) generalized negative binomial distribution and some of its applications. A brief list of authors and their works can be seen in Consul, (1989) and Johnson, Kotz and Kemp (1992) and Consul and Famoye (2006).

In this paper we have made an attempt to obtain Bayes estimator and reliability function of zero-truncated generalized Poisson distribution (ZTGPD) (1.3) for one parameter $\alpha$ when other parameter $\beta$ is assumed known in section 2 and 4. Furthermore, recurrence relations for the estimators of the parameter are also obtained in section 3. Monte Carlo simulation was performed and a comparison has been made of the Bayes estimator and reliability function of (1.3) with the corresponding maximum likelihood estimator (MLE) in section 5.

## II. BAYES ESTIMATOR OF PARAMETER OF ZERO-TRUNCATED GENERALIZED POISSON DISTRIBUTION

Let $x_1, x_2, \cdots, x_n$ be a random sample from (1.3). The likelihood function is given by

$$L(\underline{x}/\alpha,\beta)=\prod_{i=1}^{n}\left[\frac{(1+\beta x_i)^{x_i-1}}{x_i!}\right]\alpha^{\sum_{i=1}^{n}x_i}\, e^{-\left(n+\beta\sum_{i=1}^{n}x_i\right)\alpha}\,(1-e^{-\alpha})^{-n}$$

$$=C\alpha^{y}e^{-(n+\beta y)\alpha}(1-e^{-\alpha})^{-n} \tag{2.1}$$

where $C = \prod_{i=1}^{n} \left[ \frac{(1+\beta x_i)^{x_i - 1}}{x_i !} \right]$ *and* $y = \sum_{i=1}^{n} x_i$ .

When $\beta$ is known, the part of the likelihood function which is relevant to Bayesian inference on the unknown parameter $\alpha$ is

$$\alpha^y e^{-(n+\beta y)\alpha} (1-e^{-\alpha})^{-n} .$$

Suppose that the parameter $\alpha$ in (1.3) is a random variable but $\beta$ is known. Let us consider the prior distribution of $\alpha$ to be gamma distribution with known parameters *a>0* and *b>0* having density function

$$g(\alpha \mid a,b) = \frac{a^b}{\Gamma(b)} e^{-a\alpha} \alpha^{b-1}, \quad a,b > 0, \ \alpha > 0 \tag{2.2}$$

Using Bayes theorem, the posterior distribution of $\alpha$ from (2.1) and (2.2) can be shown to be

$$p(\alpha \mid y) = \frac{\alpha^{y+b-1} e^{-(n+a+\beta y)\alpha} (1-e^{-\alpha})^{-n}}{\int_0^\infty \alpha^{y+b-1} e^{-(n+a+\beta y)\alpha} (1-e^{-\alpha})^{-n} d\alpha.} \tag{2.3}$$

Under square error loss function, the Bayes estimator of $\alpha$ is given as

$$\alpha^* (y,a,n) = \int_0^\infty \alpha \ p(\alpha / y) d\alpha$$

$$= \frac{\int_0^\infty \alpha^{y+b} e^{-(n+a+\beta y)\alpha} (1-e^{-\alpha})^{-n} d\alpha}{\int_0^\infty \alpha^{y+b-1} e^{-(n+a+\beta y)\alpha} (1-e^{-\alpha})^{-n} d\alpha.} \tag{2.4}$$

Using the identity

$$(1-z)^{-n} = \sum_{k=0}^{\infty} \binom{n+k-1}{k} z^k, \ /z/ < 1$$

and the relation $\int_0^\infty e^{-at} t^{b-1} dt = \frac{\Gamma(b)}{a^b}, \ a,b > 0, \ t > 0$

where $\Gamma(b) = \int_0^\infty e^{-t} t^{b-1} dt$

we obtain,

$$\int_0^\infty \alpha^{y+b} e^{-(n+a+\beta y)\alpha} (1-e^{-\alpha})^{-n} d\alpha = \sum_{k=0}^{\infty} \binom{n+k-1}{k} \int_0^\infty e^{-(n+a+\beta y+k)\alpha} \alpha^{y+b} d\alpha$$

$$= \Gamma(y+b+1) \sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(a+\beta y+k)^{y+b+1}} \qquad (2.5)$$

and similarly

$$\int_0^\infty \alpha^{y+b-1} e^{-(n+a+\beta y)\alpha} (1-e^{-\alpha})^{-n} d\alpha = \Gamma(y+b) \sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(a+\beta y+k)^{y+b}} \qquad (2.6)$$

Substituting the values of (2.5) and (2.6) in (2.4) and using the relations $\Gamma(b+1) = b\Gamma(b)$ *and*

$$k\binom{k-1}{n-1} = n\binom{k-1}{n} + n\binom{k-1}{n-1} \qquad (2.7)$$

we get

$$\alpha^*(y,a,n) = \frac{(y+b)\sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(a+\beta y+k)^{y+b+1}}}{\sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{(a+\beta y+k)}{(a+\beta y+k)^{y+b+1}}}$$

$$= \frac{(y+b)\sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(a+\beta y+k)^{y+b+1}}}{n\sum_{k=n+1}^{\infty} \binom{k-1}{n} \frac{1}{(a+\beta y+k)^{y+b+1}} + (n+a+\beta y)\sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(a+\beta y+k)^{y+b+1}}}$$

or

$$\alpha^*(y,a,n) = \frac{(y+b)M(y,a,n)}{nM(y,a,n+1) + (n+a+\beta y)M(y,a,n)} \qquad (2.8)$$

$$y = n,\ n+1, \cdots, \qquad n = 1, 2, \cdots$$

where

$$M(y,a,n) = \sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(a+\beta y+k)^{y+b+1}} \qquad (2.9)$$

$$M(y,a,n+1) = \sum_{k=n+1}^{\infty} \binom{k-1}{n} \frac{1}{(a+\beta y+k)^{y+b+1}} \qquad (2.10)$$

After simplification (2.8) becomes

$$\alpha^*(y,a,n)=\frac{(y+b)}{\{(n+a+\beta y)+nA(y,a,n)\}};y=n,\ n+1,\ \cdots,\ n=1,\ 2\ \cdots \tag{2.11}$$

where $$A(y,a,n)=\frac{M(y,a,n+1)}{M(y,a,n)} \tag{2.12}$$

## III. RECURRENCE RELATIONS

In order to obtain a recurrence relation for $\alpha^*(y,a,n)$, first we need recurrence relations for the numbers M(y, a, n) and A(y, a, n), which are obtained by the following two lemmas:

***Lemma 1***: The numbers M(y, a, n), satisfy the recurrence relation:

$$M(y,a,n+1)=\frac{1}{n}M(y-1,a+\beta,n)-\frac{(n+a+\beta y)}{n}M(y,a,n) \tag{3.1}$$
$$y=n,\ n+1,\cdots,\quad n=1,\ 2,\ \cdots$$

with initial condition

$$M(y,a,1)=\sum_{k=1}^{\infty}\frac{1}{(a+\beta y+k)^{y+b+1}}\quad;\quad y=1,2,\cdots \tag{3.2}$$

***Proof***: From the relation (2.9), we have

$$M(y-1,\ a+\beta,\ n)=\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{1}{(a+\beta+\beta(y-1)+k)^{y+b}}$$
$$=\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{(a+\beta y+k)}{(a+\beta y+k)^{y+b+1}}$$
$$=\sum_{k=n}^{\infty}k\binom{k-1}{n-1}\frac{1}{(a+\beta y+k)^{y+b+1}}+(a+\beta y)\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{1}{(a+\beta y+k)^{y+b+1}} \tag{3.3}$$

Using the relation (2.7) then (3.3) becomes

$$M(y-1,a+\beta,n)=n\sum_{k=n+1}^{\infty}\binom{k-1}{n}\frac{1}{(a+\beta y+k)^{y+b+1}}$$
$$+(n+a+\beta y)\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{1}{(a+\beta y+k)^{y+b+1}}$$

From (2.9) and (2.10) we have,

$$M(y-1,a+\beta,n)=nM(y,a,n+1)+(n+a+\beta y)M(y,a,n) \tag{3.4}$$

from which, we have (3.1). Also from (2.9) for n = 1, we have the relation (3.2)

***Remark 1*:** Since, a is a positive integer and b > 0, from (2.9) we have

$$\begin{aligned}
M(y,a,1) &= \sum_{k=1}^{\infty}\frac{1}{(a+\beta y+k)^{y+b+1}} \\
&= \frac{1}{y+b}\sum_{k=1}^{\infty}\left\{\frac{1}{(a+\beta y+k-1)^{y+b}}-\frac{1}{(a+\beta y+k)^{y+b}}\right\} \\
&\leq \frac{1}{y+b}\sum_{m=a+\beta y+1}^{\infty}\left\{\frac{1}{(m-1)^{y+b}}-\frac{1}{m^{y+b}}\right\} \\
&= \frac{1}{(y+b)}\cdot\frac{1}{(a+\beta y)^{y+b}}
\end{aligned}$$

We also have

$$M(y,a,1)\geq\frac{1}{(a+\beta y+1)^{y+b+1}}$$

consequently the series M(y, a, 1) exists and from (3.1) by mathematical induction we conclude that the series M(y, a, n) also exists.

***Remark 2*:** Combining equations (2.11), (2.12) and (3.1) we get that

$$\begin{aligned}
A(y,a,n) &= \frac{\frac{1}{n}M(y-1,a+\beta,n)-\frac{1}{n}(n+a+\beta y)M(y,a,n)}{M(y,a,n)} \\
&= \left[\frac{M(y-1,a+\beta,n)}{M(y,a,n)}-(n+a+\beta y)\right]/n
\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}
\alpha^{*}(y,a,n) &= \frac{(y+b)M(y,a,n)}{\left[(n+a+\beta y)M(y,a,n)+n\left\{\frac{1}{n}M(y-1,a+\beta,n)-\frac{1}{n}(n+a+\beta y)M(y,a,n)\right]\right.} \\
&= \frac{(y+b)M(y,a,n)}{M(y-1,a+\beta,n)}
\end{aligned} \tag{3.6}$$

***Lemma 2*:** The numbers A(y, a, n) satisfy the recurrence relation:

$$A(y,a,n+1)=\frac{[nA(y,a,n)+(n+a+\beta y)]A(y-1,a+\beta,n)}{(n+1)A(y,a,n)}-\frac{[(n+1)+a+\beta y]}{(n+1)} \tag{3.7}$$

$$n=1,2,\cdots,\qquad y=n,n+1,\cdots$$

with initial condition

$$A(y,a,1)=\frac{M(y-1,a+\beta,1)}{M(y,a,1)} \tag{3.8}$$

***Proof*:** From the relation (2.12) and the recurrence relation (3.1), we get

$$A(y,a,n+1)=\frac{M(y,a,n+2)}{M(y,a,n+1)}$$

$$=\frac{\frac{1}{(n+1)}M(y-1,a+\beta,n+1)-\frac{1}{(n+1)}(n+1+a+\beta y)M(y,a,n+1)}{M(y,a,n+1)}$$

$$=\frac{\frac{1}{(n+1)}\left[\frac{M(y-1,a+\beta,n+1)}{M(y,a,n)}-\{(n+1)+a+\beta y\}A(y,a,n)\right]}{A(y,a,n)} \tag{3.9}$$

We also have,

$$A(y-1,a+\beta,n)=\frac{M(y-1,a+\beta,n+1)}{M(y-1,a+\beta,n)}$$

$$=\frac{M(y-1,a+\beta,n+1)/M(y,a,n)}{[nM(y,a,n+1)/M(y,a,n)]+(n+a+\beta y)}$$

From (2.12) and (3.4) we have

$$A(y-1,a+\beta,n)=\frac{M(y-1,a+\beta,n+1)/M(y,a,n)}{nA(y,a,n)+(n+a+\beta y)} \tag{3.10}$$

or

$$M(y-1,a+\beta,n+1)/M(y,a,n)=[nA(y,a,n)+(n+a+\beta y)]A(y-1,a+\beta,n) \tag{3.11}$$

Substituting (3.11) in (3.9) we obtain (3.7). Also from (3.5) for n =1, we easily get the initial condition (3.8).

***Theorem 1*:** The Bayes estimator of the parameter α satisfies the recurrence relation:

$$\alpha^*(y,a,n+1) = \frac{\left[y+b-(n+a+\beta y)\alpha^*(y,a,n)\right]\alpha^*(y-1,a+\beta,n)}{(y-1+b)-(n+a+\beta y)\alpha^*(y-1,a+\beta,n)} \tag{3.12}$$

with initial conditions

$$\alpha^*(y,a,1) = \frac{(y+b)M(y,a,1)}{M(y-1,a+\beta,1)} \tag{3.13}$$

***Proof*:** From the relation (2.11), we have

$$\alpha^*(y,a,n+1) = \frac{(y+b)}{(n+1+a+\beta y)+(n+1)A(y,a,n+1)} \tag{3.14}$$

Substituting (3.7) into (3.14) and using (2.11) we get (3.12) after some algebraic manipulations. Also from the relation (3.6) for n = 1, we easily get (3.13).

## IV. BAYES ESTIMATOR OF THE RELIABILITY FUNCTION OF ZERO-TRUNCATED GENERALIZED POISSON DISTRIBUTION

The Bayes estimator $R^*(t)$, for R (t) = P(X > t), where random variable X has the distribution (1.3), is given by

$$R^*(t) = E[R(t)/x_1,x_2,\dots,x_n]$$

$$= \frac{\int_0^\infty R(t)\alpha^{y+b-1}e^{-(n+a+\beta y)\alpha}(1-e^{-\alpha})^{-n}\,d\alpha}{\int \alpha^{y+b-1}e^{-(n+a+\beta y)\alpha}(1-e^{-\alpha})^{-n}\,d\alpha} \tag{4.1}$$

where $R(t) = \sum_{x=[t]+1}^{\infty} \frac{(1+\beta x)^{x-1}\alpha^x e^{-\alpha(1+\beta x)}(1-e^{-\alpha})^{-1}}{x!}$ (4.2)

and [t] is the integer part of t.

Making similar computations, as for $\alpha^*(y,a,n)$ we obtain

$$R^*(t) = \frac{\sum_{x=[t]+1}^{\infty} \frac{(1+\beta x)^{x-1}}{x!}\int_0^\infty e^{-(n+a+\beta y+1+\beta x)\alpha}\alpha^{y+b+x-1}(1-e^{-\alpha})^{-(n+1)}\,d\alpha}{\int_0^\infty \alpha^{y+b-1}e^{-(n+a+\beta y)\alpha}(1-e^{-\alpha})^{-n}\,d\alpha} \tag{4.3}$$

Using the identity

$$(1-z)^{-n}=\sum_{k=0}^{\infty}\binom{n+k-1}{k}z^{k},\ /z/<1$$

we obtain

$$\int_{0}^{\infty}e^{-(n+a+\beta y+1+\beta x)\alpha}\alpha^{(y+b+x-1)}(1-e^{-\alpha})^{-(n+1)}d\alpha$$

$$=\sum_{k=0}^{\infty}\binom{n+k}{k}\int_{0}^{\infty}e^{-(n+a+\beta y+1+\beta x+k)\alpha}\alpha^{y+b+x-1}d\alpha$$

$$=\Gamma(y+b+x)\sum_{k=n+1}^{\infty}\binom{k-1}{n}\frac{1}{(a+\beta y+\beta x+k)^{y+b+x}} \tag{4.4}$$

Similarly,

$$\int_{0}^{\infty}\alpha^{y+b-1}e^{-(n+a+\beta y)\alpha}(1-e^{-\alpha})^{-n}d\alpha=\Gamma(y+b)\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{1}{(a+\beta y+k)^{y+b}} \tag{4.5}$$

Using (4.4) and (4.5) in (4.3), we obtain

$$R^{*}(t)=\frac{\sum_{k=[t]+1}^{\infty}\frac{\Gamma(y+b+x)(1+\beta x)^{x-1}}{x!\,\Gamma(y+b)}\sum_{k=n+1}^{\infty}\binom{k-1}{n}\frac{1}{(a+\beta y+\beta x+k)^{y+b+x}}}{\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{1}{(a+\beta y+k)^{y+b}}} \tag{4.6}$$

Using the relation (2.7), we get

$$R^{*}(t)=\sum_{k=[t]+1}^{\infty}\frac{(1+\beta x)^{x-1}}{x!}\frac{\Gamma(y+b+x)}{\Gamma(y+b)}\frac{\sum_{k=n+1}^{\infty}\binom{k-1}{n}\frac{(a+\beta y+\beta x+k)}{(a+\beta y+\beta x+k)^{y+b+x+1}}}{\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{(a+\beta y+k)}{(a+\beta y+k)^{y+b+1}}}$$

$$=\sum_{k=[t]+1}^{\infty}\frac{(1+\beta x)^{x-1}}{x!}\frac{\Gamma(y+b+x)}{\Gamma(y+b)}\left[\frac{(n+1)\sum_{k=n+2}^{\infty}\binom{k-1}{n+1}W+(n+1+a+\beta y+\beta x)\sum_{k=n+1}^{\infty}\binom{k-1}{n}W}{n\sum_{k=n+1}^{\infty}\binom{k-1}{n}R+(n+a+\beta y)\sum_{k=n}^{\infty}\binom{k-1}{n-1}R}\right]$$

where $W=\dfrac{1}{(a+\beta y+\beta x+k)^{y+b+x+1}}$ and $R=\dfrac{1}{(a+\beta y+k)^{y+b+1}}$

$$R^{*}(t)=\sum_{k=[t]+1}^{\infty}\frac{(1+\beta x)^{x-1}}{x!}\frac{\Gamma(y+b+x)}{\Gamma(y+b)}\left[\frac{(n+1)M(y+x,a,n+2)+(n+1+a+\beta y+\beta x)M(y+x,a,n+1)}{nM(y,a,n+1)+(n+a+\beta y)M(y,a,n)}\right] \tag{4.7}$$

where the numbers $M(y,a,n)$ are given in (2.9).

## V. COMPUTER SIMULATION AND CONCLUSIONS

To compare the estimators, Monte Carlo Simulations were performed on 1000 samples for each simulation. The following steps summarize the simulation:

1. A value is generated from a gamma distribution with specified parameters a and b.
2. Based on the realization from the Gamma distribution a sample of size n=10 or 30 are generated from the zero-truncated generalized Poisson distribution with β known
3. The estimates of the parameter and reliability function are computed from the generated sample, and the estimates and their squared error losses were stored.
4. Steps 1-3 were repeated 1000 times.
5. Average values and root mean square errors (RMSE's) of the estimates are computed over the 1000 samples.

Tables 1-4 show some of the results. In comparing the estimators the root mean square error (RMSE) criterion will be used, namely the estimator with the smallest RMSE is the best estimator. The reliability function was evaluated arbitrarily at times 1, 2 and 3. Two sample size of n=10, 30 were utilized in the simulation.

**Table 1: Average values and RMSE's for the estimators of the Zero-truncated Generalized Poisson distribution**
**Gamma prior with a=1 and b=1. Sample size n=10 and β = 0.2**

| Parameter | | | | | | |
|---|---|---|---|---|---|---|
| **True value** | | **Bayes** | | **MLE** | | **RMSE Ratio** |
| | | Ave. | RMSE | Ave. | RMSE | MLE/Bayes |
| 3.7832 | | 3.7776 | 3.3190 | 3.7784 | 3.3200 | 1.0003 |
| **Reliability** | | | | | | |
| **Time** | **Exact value** | **Bayes** | | **MLE** | | **RMSE Ratio** |
| | | Ave. | RMSE | Ave. | RMSE | MLE/Bayes |
| 1 | 4.1816 | 4.1724 | 3.3230 | 4.1695 | 3.3264 | 1.0010 |
| 2 | 4.0879 | 4.0769 | 3.3392 | 4.0779 | 3.3408 | 1.0005 |
| 3 | 3.978 | 3.9681 | 3.3523 | 3.9699 | 3.3542 | 1.0006 |

**Table 2: Average values and RMSE's for the estimators of the Zero-truncated Generalized Poisson distribution**
**Gamma prior with a=1 and b=1. Sample size n=30 and β = 0.2**

| Parameter | | | | | | |
|---|---|---|---|---|---|---|
| **True value** | | **Bayes** | | **MLE** | | **RMSE Ratio** |
| | | Ave. | RMSE | Ave. | RMSE | MLE/Bayes |
| 14.8380 | | 14.8020 | 14.3255 | 14.8020<br>14.3256 | | 1.0000 |
| **Reliability** | | | | | | |
| **Time** | **Exact value** | **Bayes** | | **MLE** | | **RMSE Ratio** |
| | | Ave. | RMSE | Ave. | RMSE | MLE/Bayes |
| 1 | 15.2245 | 15.1889 | 14.3310 | 15.1882 | 14.3311 | 1.0000 |
| 2 | 15.1269 | 15.0892 | 14.3370 | 15.0890 | 14.3370 | 1.0000 |
| 3 | 15.0205 | 14.981 | 14.3420 | 14.9821 | 14.3422 | 1.0000 |

**Table 3: Average values and RMSE's for the estimators of the Zero-truncated Generalized Poisson distribution**
**Gamma prior with a=2 and b=5. Sample size n=10 and β = 0.2**

| Parameter | | | | | | |
|---|---|---|---|---|---|---|
| **True Value** | | **Bayes** | | **MLE** | | **RMSE Ratio** |
| | | Ave. | RMSE | Ave. | RMSE | MLE/Bayes |
| 1.8904 | | 1.8912 | .6665 | 1.8792 | 1.6708 | 1.0026 |
| **Reliability** | | | | | | |
| **Time** | **Exact value** | **Bayes** | | **MLE** | | **RMSE Ratio** |
| | | Ave. | RMSE | Ave. | RMSE | MLE/Bayes |
| 1 | 2.4034 | 2.3915 | 1.7026 | 2.3771 | 1.7142 | 1.0068 |
| 2 | 2.1588 | 2.1526 | 1.7305 | 2.1403 | 1.7396 | 1.0052 |
| 3 | 1.9387 | 1.9435 | 1.7244 | 1.9321 | 1.7324 | 1.0046 |

**Table 4: Average values and RMSE's for the estimators of the Zero- truncated Generalized Poisson Distribution**
**Gamma prior with a=2 and b=5. Sample Size n=30 and $\beta = 0.2$**

| Parameter | | | | | | |
|---|---|---|---|---|---|---|
| **True Value** | | **Bayes** | | **MLE** | | **RMSE Ratio** |
| | | Ave. | RMSE | Ave. | RMSE | MLE/Bayes |
| 8.2200 | | 8.1870 | 7.9306 | 8.1840 | 7.9311 | 1.0001 |
| **Reliability** | | | | | | |
| **Time** | **Exact value** | **Bayes** | | **MLE** | | **RMSE Ratio** |
| | | Ave. | RMSE | Ave. | RMSE | MLE/Bayes |
| 1 | 8.7363 | 8.6992 | 7.9502 | 8.6961 | 7.9520 | 1.0002 |
| 2 | 8.5006 | 8.4641 | 7.9642 | 8.4617 | 7.9655 | 1.0002 |
| 3 | 8.2798 | 8.2466 | 7.9619 | 8.2438 | 7.9630 | 1.0001 |

In comparing the estimators, the Bayes estimators have the smallest RMSE and are better. This is to be expected since the Bayes estimators take advantage of the known prior parameters *a* and *b*. By examining the RMSE ratios we can conclude that the estimates are sensitive to the choice of prior parameters and to sample size.

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